# High Accuracy Distance Measurement Using Frequency Comb

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***Abstract*—High precision indoor position estimation enables new opportunities for a variety of commercial, industrial and consumer applications. In this paper, we consider a phase-based method to calculate range from noisy measurements of a Frequency Comb in a multi-fading environment. It can be used to determine the range between devices for the next-generation High Accuracy Distance Measurement (HADM) protocol. We have conducted a quantitative analysis of various estimation approaches, considering both Monte-Carlo simulations of synthetic data in a variety of ranges. Moreover, we have proposed evaluation schemes for situations in which only a subset of data is available and some information may be missing, which extends existing approaches.**

***Index Terms*—BLE, Frequency Comb, HADM, WiFi**

## I. Introduction

The Global Navigation Satellite System (GNSS) is extremely effective in providing localization data [1] in open outdoor environments. Unfortunately, because of the weak signal, GNSS has an inadmissible performance indoors. Today, indoor positioning plays a major role in a number of fields [2], and measuring the distance accurately between two devices is essential for many applications. For example, it can be used to provide facility visitors with a map that shows points of interest location on their phones' screens [1]. Smart buildings can be empowered by this technology, navigation is improved, and finding items is no longer a time consuming process. Location based services, for instance, can also be enhanced by using accurate distances. It allows business owners to serve location-based advertising and content. Moreover, location services can be interleaved with other technologies, such as Internet of Things (IoT) and analytics, to trigger specific actions based on a user's location [3]. Due to the complexity of indoor spaces and topologies, it is still challenging to apply accurate, effective, and real-time positioning to indoor environments.

We address the problem of phase-based distance estimation from noisy measurements in a multi-fading environment for the next generation Bluetooth Low Energy (BLE) High Accuracy Distance Measurement (HADM) protocol. HADM is a new feature in the Bluetooth (BT) standard. It uses a CW comb to measure gain and phase in BT channels separated by 1MHz through the Quick Tone Exchange (QTE) method. The transponder locks on the phase of the signal sent by the transmitter and actively reflects it back to the transmitter, thus mimicking the working principle of the radar, as illustrated in Figure 1.

Several distance measurement techniques have been considered for the BLE protocol. The most basic solutions for this problem have been based on Received Signal Strength Indicator (RSSI) [4], where the distance is calculated based on attenuation of the transmitted signal. Applying the free-space signal propagation formula, the receiver measures the distance from the square root of the ratio of the transmitted and the received signals. However this method, although simple to implement, suffers from severe inaccuracy in multi-fading environments since the inverse square law law does not longer holds. A different, time-based approach, where the receiver calculates the distance based on propagation delay, has been implemented in a variety of communication protocols. This approach, albeit being considered very accurate and reliable, is not applicable to BLE due to its broad bandwidth requirements. Recently, there has been proposed a phase-based distance measurement technique, which will be the focus of our work. In that approach, the distance between the transmitter and the receiver is estimated from the phase difference between the received and transmitted signals across multiple frequency bands.

Linear phase difference across uniform sampled measurements corresponds to a fundamental frequency. Estimation of a frequency from noisy samples of a tone has been excessively studied in the signal processing literature. In [5] analysis is done on the base-band variant of a single tone with unknown amplitude, frequency and phase, corrupted with complex Gaussian noise. Maximum Likelihood (ML) algorithms were derived in order to estimate the unknown parameters. In particular, it was shown that the ML estimator of the frequency is maximal index of the DFT transform of

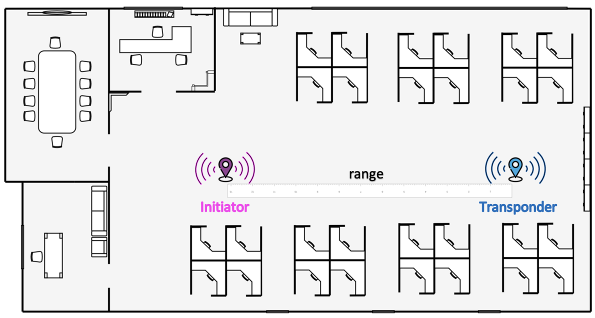



Figure 1: QTE based indoor positioning

the signal.

In many practical scenarios, including WiFi and BLE, the protocol is based on multiple carriers transmission signal that usually consists of multiple tones. MUSIC algorithm [6] is suitable to accommodate this scenario. The MUSIC algorithm detects frequencies in a signal by performing an eigenvalue decomposition on the covariance matrix of a data vector obtained from the samples of the received signal. It reduces the undesired peaks by averaging, using a frequency estimation function. The frequencies of the complex exponentials are in the location of the $p$ largest peaks in of the function. In [7], it was shown that the capabilities of the MUSIC algorithm might be enhanced by development of two techniques to allow the algorithm to operate on a single snapshot and at a much lower computational cost than previously possible. ESPRIT algorithm [8] is another technique used to determine parameters of a mixture of sinusoidals in background noise. In comparison to MUSIC, it achieves a significant reduction in computational complexity by imposing a constraint on the structure of the sensor array. Furthermore, ESPRIT can handle as many sources as MUSIC by employing overlapping subarrays.

There are also methods that base their estimation solely on the phase of the signal, thus, completely neglecting its amplitude. In [9], the authors present ways to estimate frequency by performing linear regression on the phase of the signal. In [3], FFT was applied on the Power Spectral Density (PSD) of the signal's phase. It was shown that qualitative estimation can be achieved without extra calculation and get error reduction with comparison to other measurement tools. In [4], a phase-based ranging solution was introduced for Bluetooth Low Energy (BLE) standards, in which tones where exchanged in the entire 2.4 GHz frequency band in order to overcome the multipath fading problem. Also, the effect of BLE packet spacing was analytically studied and the effect on ranging accuracy was shown on hardware implementation.

A common obstacle in the ranging problem for the mentioned methods is a multipath fading. Multipath propagation effects [10] are among the most troublesome aspects of indoor radio communications. A multipath effect occurs when the same signal arrives simultaneously from multiple directions. Since every path the signal travels has different channel characteristics, it might interfere with another copy of itself at the receiver, both in a constructive and a destructive way. When modeling such dynamic environments, several problems arise [11]. One is that a signal might have its Line-Of-Sight (LOS) blocked by some object, attenuating the most direct paths signal. Other more subtle problems are the different characteristics of fading introduced for different kinds of obstacles. Some materials are prone to reflect radio waves, while others simply attenuate it. As it is virtually impossible to identify a material and determine its properties in real time, multipath effects are better compensated for in a statistical sense. The effects should be most pronounced when using RSSI based systems, but they also might affect both Time Of Flight (ToF) [2], and phase-based ones, albeit not as greatly. Also, another problem that we might encounter is the discontinuity of the phase, since it is limited to $0-2\pi$, this adds range ambiguity and a serious challenge to range estimation. To solve the ambiguity added by definition of phase and multipath fading, phase changes of multi-frequencies are measured based on Multi-Carrier Phase Difference procedure (MCPD) in order to estimate the range [4].

In this paper we will propose and compare different methods to estimate range from a set of noisy measurements in a multipath environment. The main contribution of this work is a quantitative assessment of the various approaches both using Monte-Carlo simulations on synthetic data. Furthermore, we extend the existing approaches to scenarios where only a subset of the data is available and some information may be absent.

## II. Model Description

In this paper we consider a multiple orthogonal channel transmission protocol. The WiFi or BLE terminals, such as Access Point (AP) and Network Interface Card (NIC), transmits a single tone in every one of the channel bands. This tone is received by the initiator and reflector and transmitted back to the sensor with appropriate phase, thus resembling the operation of a radar. The noisy signal, including its multipath echos, is received by the sensor, demodulated, sampled and then DFT processed. Then, the complex number corresponding to the maximal absolute value of the DFT, is extracted and provided to the estimator. Thus, one sample of the received base-band signal at band $k$ is given by:

$$\mathsf{Y}_k = A_0 \cdot e^{\frac{j4\pi r \Delta_f k}{c}} + \sum_{m=1}^{M} A_m \cdot e^{\frac{j4\pi r_m \Delta_f k}{c}} + \mathsf{Z}_k, \quad (1)$$

where:

- $A_0$ - the complex amplitude of the LOS signal;
- $r$ - the range between terminals;
- $\Delta_f$ - spacing between frequency bands;
- $c$ - speed of light;
- $M$ - number of multipaths, unknown integer;
- $\{r_m\}_{m=1}^{M}$ - multipaths lengths, unknown parameters but strictly greater than $r$;
- $\{A_m\}_{m=1}^{M}$ - complex amplitudes of the multipath signals;
- $\{\mathsf{Z}_k\}_{k=1}^{K}$ - i.i.d. Complex Gaussian random variables with zero mean and variance $\sigma^2$, i.e., $\mathsf{Z}_k \sim \mathcal{CN}(0, \sigma^2)$.

Our goal in this work is to find an optimal estimator of $r$ based on the measurements vector $(\mathsf{Y}_1, \mathsf{Y}_2, \ldots, \mathsf{Y}_K)$ subject the criterion of minimal Mean Square Error (MSE).

Furthermore, we intend to test a variety of solution approaches to range estimation with incomplete data, and to evaluate the correlation between resolution and accuracy.

In the following subsection we derive a processing-independent, analog resolution of the presented model.

### A. Analog Resolution

Assume noiseless and echo-less environment with target located at location $r$. The received signal at frequency band $k$ is given by:

$$\mathsf{Y}_k = A_0 \cdot e^{\frac{j4\pi r \Delta_f k}{c}}. \tag{2}$$

Now, consider the sum of $Y_k$ which is equivalent to looking on the zero index of the DFT.

$$\mathsf{S}_N(r) = \sum_{k=0}^{N-1} Y_k = A_0 \frac{e^{\frac{j2\pi r \Delta_f N}{c}}}{e^{\frac{j2\pi r \Delta_f}{c}}} \frac{\sin\left(\frac{2\pi r \Delta_f N}{c}\right)}{\sin\left(\frac{2\pi r \Delta_f}{c}\right)}. \tag{3}$$

The relative intensity is defined as

$$g(r) \triangleq \left| \frac{S_N\ (r)}{A_0} \right| = \left| \frac{\sin\left(\frac{2\pi r \Delta_f N}{c}\right)}{\sin\left(\frac{2\pi r \Delta_f}{c}\right)} \right|. \tag{4}$$

Note that $g(r) \leq N$. Analog resolution is defined as half power intensity, i.e.,

$$\delta r_{analog} \triangleq \min_{r \geq 0} \left\{ r : g(r) = \frac{N}{\sqrt{2}} \right\}. \tag{5}$$

Figure 2 shows $g(r)$ for $\Delta_f = 312.5$ $[KHz]$ and $N = 245$, where the highlighted point represents the calculated analog resolution, $\delta r_{analog} = 0.00708$. For a general $N$, $\delta r_{analog}$ can be approximated using

$$\delta r_{analog} \approx \frac{c}{\Delta_f} \cdot \frac{0.4252}{N^2}. \tag{6}$$

We emphasize that the meaning of the analog resolution here is that it is not possible to separate between two targets in a distance smaller than $\delta r_{analog}$, as if they were in the same position. Of course, we can get a better accuracy by zero padding under high SNR conditions.

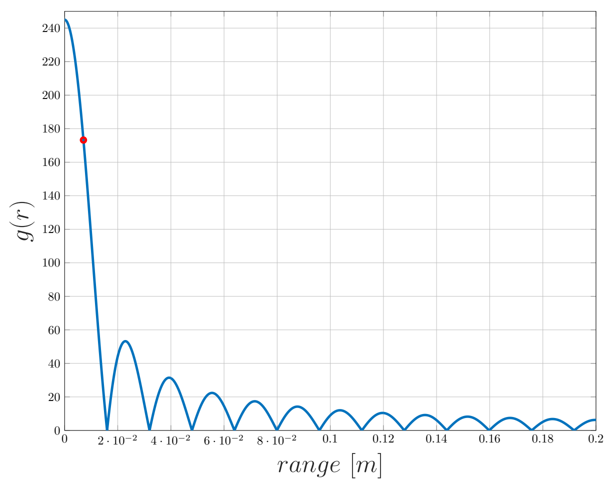



Figure 2: Analog resolution

## III. Methods

In this section, we will briefly describe a number of algorithms that will be evaluated and compared for the given range estimation problem. We will also elaborate here regarding possible extension of the existing methods to handle missing information. The various methods we present are mainly divided into two groups: those based solely on the phase of the received tones, which will be described first; and those which utilize the full complex measurements vector.

In the simulations shown in the following figures of this section, the range is chosen to be $10 \cdot Dr$ where $\Delta_f = 312.5$ $[kHz]$, $Dr = \frac{c}{2 \cdot \Delta_f \cdot N}$ and $N = 245$. Also, the high SNR signal has a noise variance $\sigma = 0.01$ and multi-path amplitude $0.1$, whereas the low SNR signal has variance $\sigma = 2$ and multi-path amplitude $0.5$.

### A. Principle of Distance estimation using Multi Carrier Phase Difference (MCPD)

The phase-shift introduced by a pure LOS component of the signal in (13) is a linear function of both frequency index $k$, and range $r$, i.e.,

$$\phi(k, r) = \frac{4\pi r \Delta_f k}{c} \mod 2\pi. \tag{7}$$

A typical graph of (7) is illustrated in Figure 3.

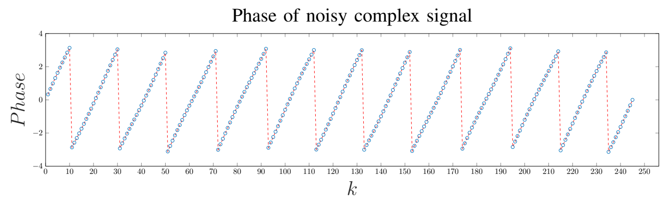



Figure 3: Phase with one multipath fading

From the relation in (7) we observe that by measuring the slope of the unwrapped phase, denoted as $\phi^{UW}(\cdot, r)$, as a function of the frequency index we can ideally have a perfect estimation of the distance between the devices.

In order to solve the half-wavelength ambiguity with respect to $r$, we measure the phase shift at at least two distinct tones:

$$\Delta\phi[k_0, k_1] \triangleq \phi^{UW}(k_1, r) - \phi^{UW}(k_0, r) = \frac{4\pi r \Delta_f (k_1 - k_0)}{c}. \tag{8}$$

Since $\Delta_f$, $c$, $k_j$ are known parameters, the distance estimation becomes [4]:

$$\hat{r} = \frac{c}{4\pi \Delta f} \frac{\Delta\phi[k_0, k_1]}{k_1 - k_0} \mod \frac{c}{4\Delta_f}. \tag{9}$$

The aforementioned procedure can be further extended to consolidate $K$ tones. We denote $\Delta\phi[k] \triangleq \Delta\phi[k-1, k]$ as the phase difference between the $k$-th and the $k-1$ tones, to obtain the following vector of distance estimators:

$$\begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \\ \vdots \\ \hat{r}_{K-1} \end{pmatrix} = \frac{c}{4\pi\Delta_f} \times \begin{pmatrix} \hat{\Delta}\phi[1] \\ \hat{\Delta}\phi[2] \\ \vdots \\ \hat{\Delta}\phi[K-1] \end{pmatrix} \quad (10)$$

Equation (10) contains a vector of phase-based estimators. Although taking the mean from (10) is the most reasonable method to obtain the combined estimator, we have observed that taking the median usually outperforms the expectation approach for real data measurement, this probably is due to numerous outliers present in the data. The estimation scheme is outlined in Algorithm 1, where the median range of the ranges vector in (10) gave a better estimation as it reduces the effect of outliers.

**Algorithm 1** Multi Carrier Phase Derivative Algorithm

```
function [range] = MCPD(angle,Df)
c = 299792458;
dvec = diff(angle); % Vector of phase differences
rvec = c/(4*Pi*Df); % Vector of range estimators
range = median(rvec); % Find the median (eliminates outliers);
end
```

This estimation scheme can be extended to incorporate missing data scenarios by taking into account the index differences in the denominator of (9) while evaluating (10).

### B. Linear Phase Regression

Here we present a Least Squares approach to estimate the range in (7). Since each discontinuity in the phase function has a $2\pi$ period, then in order to calculate the slope and get the range $r$, we can do phase unwrapping by adding $2\pi$ in every discontinuity, and then do a linear regression which fits the unwrapped phase to a linear curve. An example is shown in Figure 4, where specifically in this example we used the high SNR noise parameters described in the beginning of the section, but we used $\sigma = 0.5$ instead of $\sigma = 0.01$ to emphasize the effect of the noise on the linear regression fitted slope.

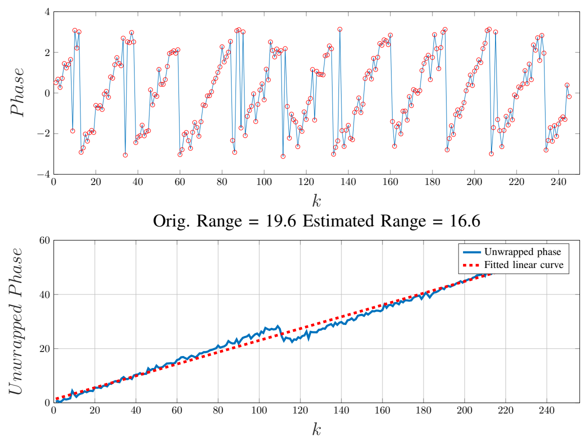



Figure 4: Phase unwrap example

If there is a missing data set among a band of channels, we suggest that the linear regression should be done assuming that the missing channels are ignored.

### C. DFT on phase of the complex signal

The periodic nature of the signal's phase, as can be observed in Figure 3, can be utilized to extract the slope by calculating the DFT on signal's phase, i.e. $\{\sphericalangle \mathsf{Y}_k\}_{k=1}^{K}$. The index of the absolute maximum value of the Fourier transform will give us the fundamental frequency of the signal's phase. The fundamental frequency has one-to-one correspondence with the range.

Missing measurements can be replaced with zeros without affecting the proposed here estimation scheme, and has a similar interpretation to the regular zero padding.

### D. DFT of the complex signal

As can be interpreted from equation (13), the LOS component of the sampled signal is given by:

$$s_k = A_0 \cdot e^{\frac{j4\pi r\Delta_f k}{c}}. \quad (11)$$

It was shown in [5] that the ML frequency estimator of a signal contaminated in white Gaussian noise is related to the maximum index of the corresponding DFT of the signal defined as:

$$DFT\left(\{\mathsf{Y}_k\}_{k=1}^{K}\right)[n] = \sum_{k=1}^{K} \mathsf{Y}_k \cdot e^{-\frac{j2\pi nk}{K}}. \quad (12)$$

Thus, we expect to observe the highest peak in the closest integer that satisfies $\frac{n}{K} = \frac{2r\Delta_f}{c}$.

Figure 5 shows a comparison between methods mentioned above by applying them on a high SNR signal and a low SNR signal. The DFT on the signal gives more accurate results, regardless of the noise level, making it the better choice here.

Extension of the proposed scheme here to handle incomplete measurements can be performed in a similar manner to the one described for applying the DFT on signal's phase.

### E. MUSIC algorithm

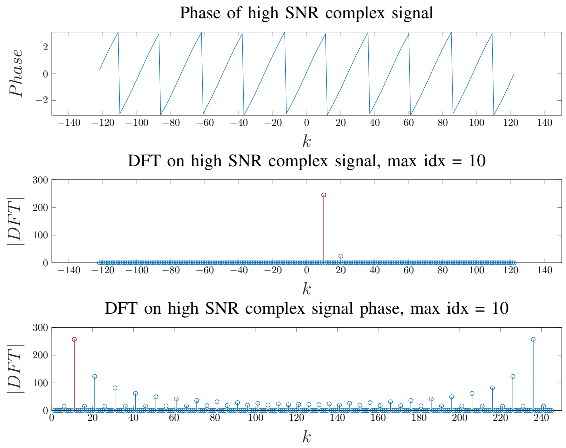
The MUltiple SIgnal Classification (MUSIC) algorithm is a frequency estimation technique [6] in which we assume that $\mathsf{Y}_n$ is a random process that consists of $p$ complex exponentials in white noise $\mathsf{Z}_n$ with variance $\sigma_z^2$:

$$\mathsf{Y}_n = \sum_{i=1}^{p} A_i \cdot e^{jn\omega_i} + \mathsf{Z}_n, \quad (13)$$

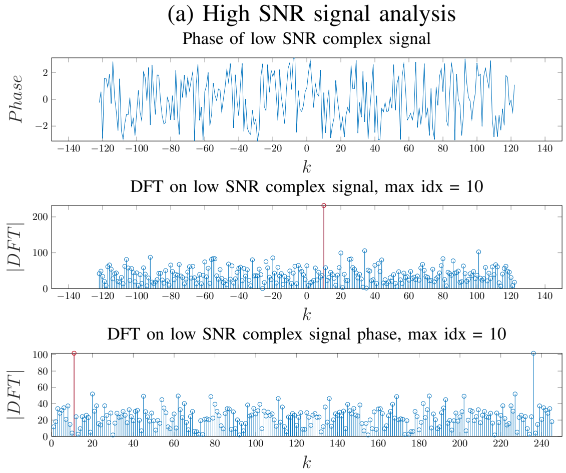
Let $R_x$ be the $M \times M$ autocorrelation matrix of $\mathsf{Y}_n$, with $M > p+1$. Then, we calculate the eigenvectors $\nu_1 \dots \nu_p, \nu_{p+1}, \dots, \nu_M$ of $R_x$, where $\nu_1$ corresponds to the largest eigenvalue of $R_x$, $\nu_2$ correspond to the second largest eigenvalue, etc.

The MUSIC algorithm tries to reduce the undesired peaks by averaging, using the frequency estimation function:

$$\hat{P}_{MU}(e^{j\omega}) = \frac{1}{\sum_{i=p+1}^{M} \mid \mathbf{e}^H \nu_i \mid^2}, \quad (14)$$




(a) High SNR signal analysis




(b) Low SNR signal analysis

Figure 5: DFT on complex signal and its phase

where $H$ refers to the hermitian conjugate, and the vector $\mathbf{e}$ is defined as follows:

$$\mathbf{e} = \begin{pmatrix} 1 & e^{j\omega} & e^{2j\omega} & \dots & e^{j(M-1)\omega} \end{pmatrix}^T. \tag{15}$$

According to the frequency estimation function, the frequencies of the complex exponentials are in the location of the $p$ largest peaks in $P_{MU}(e^{j\omega})$.

**Algorithm 2** The MUSIC Algorithm

```
function Px = music(x,p,M)
if (M < p+1 || length(x) < M)
error('Size of R is inappropriate')
R = convar(x,M);
[v,d] = eig(R);
[y,i] = sort(diag(d));
Px = 0;
for j = 1 : M-p
Px = Px+abs(fft(v(:,i(j)),1024));
Px = -20 * log10(Px);
end
```

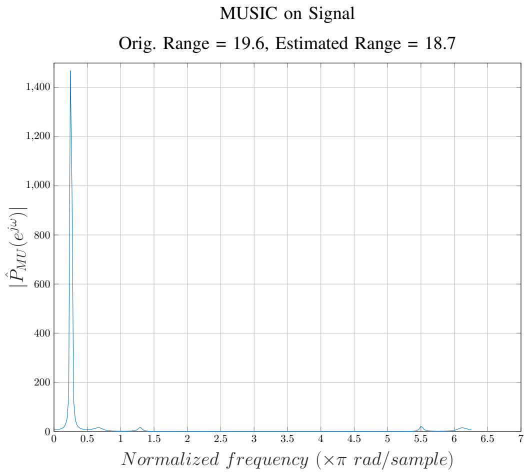



Figure 6: Simulation of the MUSIC algorithm

In Figure 6 we can see a definite peak which indicates the "normalized frequency" of the frequency-sampled vector, which is denoted $\bar{\omega}_0$. Using (9) we can derive the range as shown in

$$\hat{r} = \frac{c}{4\pi\Delta f}\bar{\omega}_0. \tag{16}$$

Note that the simulation was held on the high SNR signal with parameters described in the beginning of the section.

## IV. Performance Evaluation

In this section, the methods mentioned in Section III are evaluated and compared for various channel scenarios. Data for this simulation was generated based on the synthetic model described by (13). We use MSE as the performance metric in this section.

We start by comparing the performance of various algorithms versus SNR. The frequency estimation model described here is a nonlinear estimation and therefore threshold effects are expected. That is, for SNR values beneath a certain threshold there is usually a rapid increase in MSE as SNR further decreases. The output of the Monte Carlo experiment for fixed parameters of the LOS signal and its multipath echoes is shown in Figure 7.

Our first observation from Figure 7 is that the DFT on signal method outperforms all other in terms of MSE. Furthermore, since systems does not usually operate below the threshold level, we choose SNR level of 20 dB as a reference point for subsequent calculations.

Another valuable insight added to the graph is the Cramér–Rao Lower Bound (CRLB), which can be derived in a similar manner as in [5], and is given by

$$MSE \geq CRLB = \frac{3c^2\sigma^2}{4A_0^2\pi^2\Delta_f^2 N(N^2-1)}. \tag{17}$$

Furthermore, it is quite interesting to analyze how the behavior changes when there are missing bands in the received signal. We present the Monte Carlo results in Figure 8 for this circumstance, where the bands $(20:40)$ are excluded from total frequency band $[1:245]$.

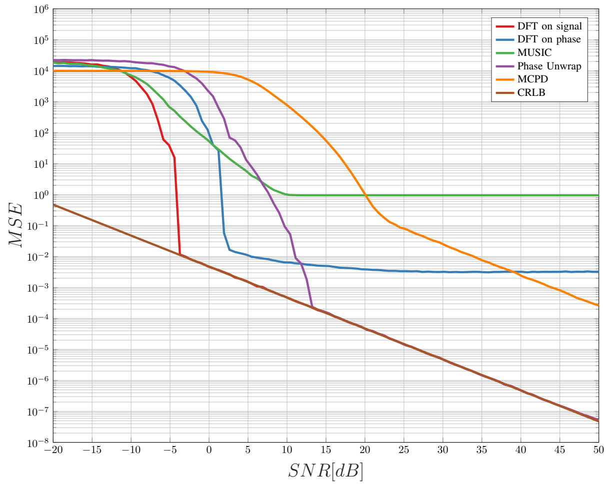



Figure 7: MSE versus SNR

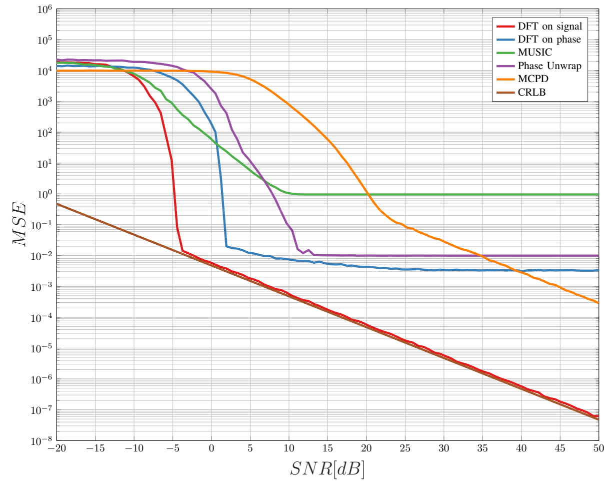



Figure 8: MSE versus SNR for a signal with missing data

We can see from this behavior that a missing band of channels can still produce reasonable results that are consistent with a full signal analysis. We also conducted a simulation showing the error for different multipath amplitude ratios relative to the original signal, while fixing $SNR = 20\,\text{dB}$. Figure 9 shows the different methods' immunity to multipath. We conclude that multipath signal amplitude slightly affects DFT and phase unwrap methods, while it significantly affects MUSIC. Also we observe the dominance of the DFT on signal method, which is able to distinguish the true range for all practical values of the multipath amplitude.

## V. Conclusion

Using a phase-based approach, a quantitative examination of range estimation methods utilizing synthetic data has been conducted in this work. We conducted the tests on a high and low SNR signal, assuming a multipath environment. We concluded that the DFT on signal method produced the best results as measured by MSE versus SNR in both scenarios with and without missing data sets.

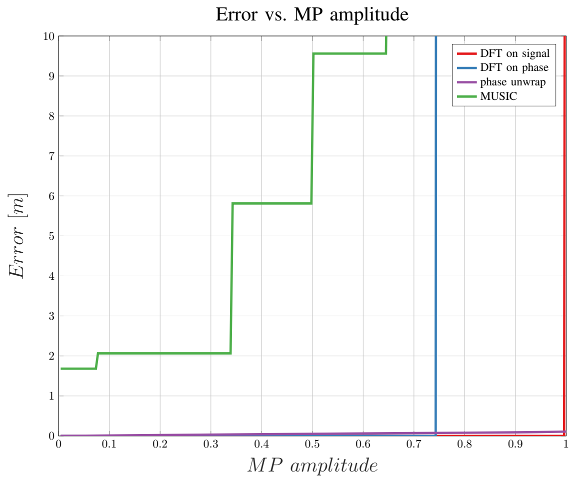



Figure 9: Error for different multipath amplitude ratios

## VI. Acknowledgements

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